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SOLUTION BY THE PROPOSER.

Let C be the third vertex. By hypothesis $CAE < CBD$ and $BAE < ABD$. A point F can then be found on CD such that $DBF = CAE$. Let BF cut AE in H . AHF and BDF are similar, having equal angles at A and B and the same angle at F . Therefore,

$$AH : BD = AF : BF. \quad (1)$$

Also, since $ABD > BAE$, $ABF > BAF$. Consequently,

$$AF > BF. \quad (2)$$

From (1) and (2), it follows that $AH > BD$. Therefore, $AE > AH > BD$, which was to be proved.

COROLLARY: If AE and BD are equal and divide their angles in the same ratio, the triangle is isosceles.

For, if the angles A and B were not equal the parts of one would be respectively less than the corresponding parts of the other and AE and BD would be unequal, which is contrary to hypothesis.

In particular, if the bisectors of two angles of a triangle are equal, the triangle is isosceles.

2736 [December, 1918]. Proposed by M. COHEN, Freshman, Johns Hopkins University.

Prove by elementary geometry that the orthocenter, the centroid, and the circumcenter of a triangle lie on a line (the Euler line), and that the centroid lies between the other two and is twice as far from the orthocenter as from the circumcenter.

SOLUTION BY J. L. RILEY, Stephenville, Texas.

Let ABC be the triangle under consideration; O and G the circumcenter and centroid, BE and CF perpendicular, respectively, to AC and AB . Let mid-point of AC be B' .

Produce OG to meet the altitude BE at K . The triangles OGB' and KGB are similar, for OB' is parallel to BK , since each is perpendicular to AC . Then $OG : GK = B'G : GB = 1 : 2$ and hence, $GK = 2 OG$.

If OG is produced to meet the altitude CF at K' , it follows in the same way that $GK' = 2 OG$. Therefore, $GK' = GK$ and K' coincides with K . Hence BE and CF meet at K and K is the orthocenter. Hence, circumcenter, centroid, and orthocenter lie on the same line.

Also solved by H. L. OLSON, C. P. SOUSLEY, and the Proposer.

2737 [January, 1919]. Proposed by C. N. SCHMALL, New York City.

Employing Maclaurin's theorem, or otherwise, expand the following three functions (1) $e^{\tan^{-1} x}$ as far as x^6 ; (2) $e^{\sin x}$ as far as x^5 ; and (3) $\tan x$ as far as x^9 .

SOLUTION BY ELMER LATSHAW, West Philadelphia, Pennsylvania.

The successive differentiation required by Maclaurin's theorem in the development of the given functions is long and laborious, but the required developments may be obtained by comparing the derivative of the function with the function itself.

Assume

$$e^{\tan^{-1} x} = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots + a_6x^6 + \dots \quad (1)$$

Differentiating both sides of (1),

$$\begin{aligned} e^{\tan^{-1} x} \frac{1}{1+x^2} &= a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 + \dots + 6a_6x^5 + \dots \\ &= (a_0 + a_1x + a_2x^2 + \dots + a_6x^6 + \dots)(1 - x^2 + x^4 - x^6 + \dots). \end{aligned}$$

Equating coefficients of like powers of x ,

$$a_1 = a_0, \quad 2a_2 = a_1, \quad 3a_3 = a_2 - a_0, \quad 4a_4 = a_3 - a_1, \quad 5a_5 = a_4 - a_2 + a_0, \quad 6a_6 = a_5 - a_3 + a_1.$$

Equation (1) by making $x = 0$ gives $a_0 = 1$ and the preceding equations give

$$a_1 = 1, \quad a_2 = \frac{1}{2}, \quad a_3 = -\frac{1}{6}, \quad a_4 = -\frac{7}{24}, \quad a_5 = \frac{1}{24}, \quad a_6 = \frac{29}{144}.$$

Hence,

$$e^{\tan^{-1} x} = 1 + x + \frac{x^2}{2} - \frac{x^3}{6} - \frac{7x^4}{24} + \frac{x^5}{24} + \frac{29x^6}{144} - \dots$$